

GROUNDS FOR QUANTUM GEOMETRODYNAMICS IN AN EXTENDED PHASE SPACE AND ITS COSMOLOGICAL CONSEQUENCES

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Abstract

Quantum geometrodynamics (QGD) in extended phase space (EPS) essentially distinguished from the Wheeler – DeWitt QGD is proposed. The grounds for constructing a new version of quantum geometrodynamics are briefly discussed. The main part in the proposed version of QGD is given to the Schrödinger equation for a wave function of the Universe. The Schrödinger equation carries information about a chosen gauge condition which fixes a reference system. The reference system is represented by a continual medium that can be called “the gravitational vacuum condensate”. A solution to the Schrödinger equation contains information about the integrated system “a physical object + observation means (the gravitational vacuum condensate)”. It may be demonstrated that the gravitational vacuum condensate appears to be a cosmological evolution factor.

In this talk I briefly describe the results of the work done by G. M. Vereshkov, V. A. Savchenko and me[1]. Our aim was to explore the possibility of constructing quantum geometrodynamics of a closed universe by a strict mathematical method without using any assumption not permitting detailed mathematical proofs. The proposed version of quantum geometrodynamics turns out to be a gauge-noninvariant theory *radically* distinguished from the Wheeler – DeWitt QGD by its content. The foundations for this investigation lay in the peculiarities of the Wheeler – DeWitt QGD among which worth mentioning are mathematical problems of gauge invariance and problems of interpretation.

One possible starting point for constructing QGD is to introduce the Batalin – Fradkin – Vilkovisky (BFV) transition amplitude

$$\langle f|i \rangle = \int D\mu \exp (iS_{eff}), \quad (1)$$

where S_{eff} is a BRST-invariant effective action, $D\mu$ is a measure in extended phase space[2]. Independence of the amplitude (1) on a gauge condition is ensured by asymptotic boundary conditions. However, there is no asymptotic states in a closed universe and an observer cannot be removed to infinity from the investigated object which is the Universe as a whole. So appealing to asymptotic boundary conditions does not seem to be justified when constructing QGD of a closed universe. As for the transition amplitude (1) considered without the asymptotic boundary conditions, there is no strict mathematical way to prove its gauge invariance. On the contrary, it may be demonstrated by explicit calculations that the amplitude (1) inevitably contains gauge-noninvariant effects (see [1]).

Another way is based on the Wheeler – DeWitt equation[3]. Let us emphasize that the Wheeler – DeWitt equation is not deducible by correct mathematical methods from a path integral or somehow else: it can be just postulated. The principle of gauge invariance is commonly thought to be a motivation for postulating the Wheeler – DeWitt equation. On the other hand, the Wheeler – DeWitt equation is known to be noninvariant under choice of a gauge variable, the lapse function N being usually considered as such a variable[4, 5]. However, the transition to another gauge variable is formally equivalent to imposing a new gauge condition, and vice versa. The latter reflects an obvious fact that the choice of gauge variables and the choice of gauge conditions have an unified interpretation: they both determine equations for the metric components $g_{0\mu}$, fixing a reference system. So, as a matter of fact, the parametrisation noninvariance of the Wheeler – DeWitt equation is ill-hidden gauge noninvariance.

It is well-known that in the Wheeler – DeWitt theory there is no quantum evolution of state vector in time. A wave function satisfying the Wheeler – DeWitt equation describes the past of the Universe as well as its future with all observers being inside the Universe in different stages of its evolution, and all observations to be made by these observers. Thus the Wheeler – DeWitt theory does not use the postulate about the reduction of a wave packet. One cannot appeal to the Copenhagen interpretation of quantum theory within the limits of the Wheeler – DeWitt QGD. One has to turn to the many-worlds interpretation of the wave function proposed by Everett[6] and applied to QGD by Wheeler[7]. The wave function of the Universe satisfying the Wheeler – DeWitt equation and certain boundary conditions is thought to be a branch of a many-worlds wave function that corresponds to a certain universe; other branches being selected by other boundary conditions. So the information about the continuous reduction of the wave function in the process of evolution of the Universe including certain observers inside is contained in the boundary conditions for the wave function only.

Bearing in mind all the mentioned above, we propose to investigate a more general theory. The features of the theory are:

- A basic equation for a wave function of the Universe is derived by a well-defined mathematical procedure.
- The assumption about asymptotic states is not taken into account.
- The theory admits of Copenhagen interpretation.

To illustrate our results it is convenient to consider the Bianchi IX model for its mathematical simplicity and physical meaningfulness. I shall remind that the interval in the Bianchi IX model looks like

$$ds^2 = N^2(t) dt^2 - \eta_{ab}(t) e_i^a e_k^b dx^i dx^k; \quad (2)$$

$$\eta_{ab}(t) = \text{diag} \left(a^2(t), b^2(t), c^2(t) \right). \quad (3)$$

We use the parametrization

$$a = \frac{1}{2} r \exp \left[\frac{1}{2} (\sqrt{3} \varphi + \chi) \right]; \quad b = \frac{1}{2} r \exp \left[\frac{1}{2} (-\sqrt{3} \varphi + \chi) \right]; \quad c = \frac{1}{2} r \exp (-\chi); \quad (4)$$

$$Q^a = (q, \varphi, \chi, \phi, \dots); \quad q = 2 \ln r; \quad \zeta(\mu, Q) = \ln \frac{r^3}{N}, \quad (5)$$

where ϕ stands for scalar fields, $\zeta(\mu, Q)$ is an arbitrary function defining a gauge variable μ through the lapse function N . The Bianchi IX model can be considered as a model of a Friedman – Robertson – Walker closed universe with $r(t)$ being a scale factor, on which a transversal nonlinear gravitational wave $\varphi(t), \chi(t)$ is superposed.

The derivation of an equation for a wave function of the Universe implies going over to a path integral with the effective action in a Lagrange form. Since the algebra of transformations generated by constraints is closed for the model, the transition amplitude (1) can be reduced to the path integral over extended configurational space involving ghost and gauge variables with the Faddeev – Popov effective action

$$S_{eff} = \int dt \left\{ \frac{1}{2} \exp [\zeta(\mu, Q^a)] \gamma_{ab} \dot{Q}^a \dot{Q}^b - \exp [-\zeta(\mu, Q^a)] U(Q^a) + \right. \\ \left. + \lambda (\dot{\mu} - f_{,a} \dot{Q}^a) + \frac{i}{\zeta_{,\mu}} \dot{\theta} \dot{\theta} \right\}. \quad (6)$$

We confine attention to the special class of gauges not depending on time

$$\mu = f(Q) + k; \quad k = \text{const}, \quad (7)$$

or, in a differential form,

$$\dot{\mu} = f_{,a} \dot{Q}^a, \quad f_{,a} \stackrel{def}{=} \frac{\partial f}{\partial Q^a}; \quad (8)$$

$\zeta_{,\mu} = \partial\zeta(\mu, Q)/\partial\mu$; $\theta, \bar{\theta}$ are the Faddeev – Popov ghosts after replacement $\bar{\theta} \rightarrow -i\bar{\theta}$; indices a, b, \dots are raised and lowered with the “metric”

$$\gamma_{ab} = \text{diag}(-1, 1, 1, 1, \dots); \quad (9)$$

$$U(Q) = e^{2q} U_g(\varphi, \chi) + e^{3q} U_s(\phi), \quad (10)$$

$$\begin{aligned} U_g(\varphi, \chi) = & \frac{2}{3} \left\{ \exp \left[2 \left(\sqrt{3} \varphi + \chi \right) \right] + \exp \left[2 \left(-\sqrt{3} \varphi + \chi \right) \right] + \exp(-4\chi) - \right. \\ & \left. - 2 \exp \left[- \left(\sqrt{3} \varphi + \chi \right) \right] - 2 \exp \left(\sqrt{3} \varphi - \chi \right) - 2 \exp(2\chi) \right\}. \end{aligned} \quad (11)$$

Variation of the action (6) yields a Lagrangian set of equations mathematically equivalent to canonical equations in EPS. This set of equations can be called *conditionally-classical* for the presence of Grassmannian variables. It is gauge-noninvariant.

Any gauge condition fixes a reference system, the latter representing the observer in the theory of gravity. So the action (6) describes the integrated system “the physical object + observation means”. According to Landau and Lifshitz[8], a continual medium with broken symmetry under diffeomorphism group transformation must be considered as a reference system in the theory of gravity. Inside the medium a periodic process is going, its characteristic being used for choosing metric measurements standards. Let us note that within the limits of the classical theory one is not able to point to an object with the above properties. However, quantum theory gives us the notion about such an object – it is the vacuum condensate. Thus the gauge-fixing term in (6) corresponds a specific subsystem being referred to as “the gravitational vacuum condensate”. The investigation of the conditionally-classical set of equations reveals the existence of a conserved quantity E describing the subsystem. As a result, the Hamiltonian constraint $H_{ph} = 0$ of general relativity is replaced by the constraint $H = E$, where H_{ph} is a Hamiltonian of gravitational and matter fields, H is a Hamiltonian in EPS.

The latter means that a Hamiltonian spectrum in the appropriate quantum theory is not limited by the unique zero eigenvalue. So the main part in this version of QGD is given to *the Schrödinger equation in extended configurational space* for a wave function of the Universe, and finding a spectrum of E becomes one of the main tasks of quantum geometrodynamics in EPS. We would like to emphasize that the problem of time, so typical of the Wheeler – DeWitt QGD, does not arise here. Moreover, one gains the opportunity to appeal to the Copenhagen interpretation.

The Schrödinger equation derived from the path integral with the effective action (6) by the standard method[9] originated from Feinman reads

$$i \frac{\partial \Psi(Q^a, \mu, \theta, \bar{\theta}; t)}{\partial t} = H \Psi(Q^a, \mu, \theta, \bar{\theta}; t), \quad (12)$$

where

$$H = -i \zeta_{,\mu} \frac{\partial}{\partial \theta} \frac{\partial}{\partial \theta} - \frac{1}{2M} \frac{\partial}{\partial Q^\alpha} M G^{\alpha\beta} \frac{\partial}{\partial Q^\beta} + e^{-\zeta} (U - V); \quad (13)$$

$$M = \text{const} \cdot \zeta_{,\mu} \exp \left(\frac{K+3}{2} \zeta \right); \quad (14)$$

$$\begin{aligned} V = & -\frac{3}{12} \frac{(\zeta_{,\mu})^a (\zeta_{,\mu})_a}{\zeta_{,\mu}^2} + \frac{(\zeta_{,\mu})_a^a}{3\zeta_{,\mu}} + \frac{K+1}{6\zeta_{,\mu}} \zeta_a (\zeta_{,\mu})^a + \\ & + \frac{1}{24} (K^2 + 3K + 2) \zeta_a \zeta^a + \frac{K+2}{6} \zeta_a^a, \end{aligned} \quad (15)$$

$$\zeta_a = \frac{\partial \zeta}{\partial Q^a} + f_{,a} \frac{\partial \zeta}{\partial \mu}; \quad G^{\alpha\beta} = e^{-\zeta} \begin{pmatrix} f_{,a} f^{,a} & f^{,a} \\ f^{,a} & \gamma^{ab} \end{pmatrix}, \quad (16)$$

$\alpha = (0, a)$, $Q^0 = \mu$, K is a number of scalar fields included in the model; the wave function is defined on extended configurational space with the coordinates Q^a , μ , θ , $\bar{\theta}$.

It is worth noting that no ill-definite mathematical expression arises when deriving Eq. (12). It is due to using the nondegenerate conditionally-classical set of equations to approximate the path integral instead of degenerate gauge-invariant equations. Alternatively, Eq. (12) can be obtained from quantum canonical equations in EPS.

The general solution to the Schrödinger equation (12) has the following structure:

$$\Psi(Q^a, Q^0, \theta, \bar{\theta}; t) = \int \Psi(Q^a) \exp(-iEt) (\bar{\theta} + i\theta) \delta(\mu - f(Q^a) - k) dE dk. \quad (17)$$

where $\Psi(Q^a)$ is a solution to the stationary equation

$$H^0 \Psi(Q^a) = E \Psi(Q^a), \quad (18)$$

$$H^0 = \left[-\frac{1}{2M} \frac{\partial}{\partial Q^a} M e^{-\zeta} \gamma^{ab} \frac{\partial}{\partial Q^b} + e^{-\zeta} (U - V) \right] \Big|_{\mu=f(Q^a)+k}. \quad (19)$$

The wave function (17) carries the information on 1) a physical object, 2) observation means (a gravitational vacuum condensate), 3) correlations between the physical object and observation means. Observation means are represented by the factored part of the wave function – by the δ -function of a gauge and by the ghosts; the physical object is described by the function $\Psi(Q^a)$; the correlations are manifested in the effective potential V and in the spectrum E , the gravitational vacuum condensate thus being a cosmological evolution factor. The dependence of the wave function (17) on ghosts is determined by the demand of norm positivity.

The question remains if it possible to go over to some particular solution satisfying the Wheeler – DeWitt equation from the general solution (17). To do it, one has to eliminate correlations between the properties of the physical object and those of observation means.

Then, one puts $E = 0$ and fixes the gauge $\mu = k$, making use of the formal possibility to go over to any given gauge by means of transformation of the parametrization function ζ . In this case under some limitations on the measure (14) one gets that the physical part of the wave function $\Psi(Q^a)$ satisfies the Wheeler – DeWitt equation. However, it is of importance to emphasize that the correlations between the physical object and observation means cannot be eliminated completely: the information about a chosen reference system is contained in the parametrization function and the parametrization function essentially determines the effective potential V . The same picture arises when one obtains the Wheeler – DeWitt equation as a corollary of the superselection rules for BRST- and anti-BRST-invariant quantum states.

In conclusion I shall describe the role of the gravitational vacuum condensate as a cosmological evolution factor. As I mentioned above, the gravitational vacuum condensate is a continual medium, a state equation of the medium depending on a chosen gauge condition. The state of the condensate is characterized by the parameter E , and the relation between E and other parameters of the theory determines a cosmological scenario. For example, in a simplified model with one of two gravitational waves, $\varphi(t)$, being frozen out, taking the parametrization function and the gauge condition to be $\zeta(\mu, Q^a) = \mu = k$, one obtains an ultrastiff state equation of the condensate:

$$p = \epsilon. \tag{20}$$

The scalar field ϕ and the gravitational vacuum condensate together form the two-component medium with a positive or negative energy density depending on the parameter E . A cosmological evolution scenario may contain the following phenomena[1]:

- cosmological expansion and contraction of space;
- cosmological singularity;
- compactification of space dimensions;
- asymptotically stationary space of less dimensions;
- inflation of the Universe.

The goal of the future investigation is to work out a full cosmological scenario, in the sense as it was understood by Grishchuk and Zeldovich[10], based on the proposed version of quantum geometrodynamics. After Grishchuk and Zeldovich we think that the Universe was created from the state “Nothing” where there exist neither space with its geometry nor time. In this state there is no gravitational wave or matter field, and no gravitational vacuum condensate either. After creation, however, the Universe occurs in a state with broken symmetry under diffeomorphism group, and the presence of the gravitational vacuum condensate is a characteristic feature of this picture. A state spectrum can be found by solving a Schrödinger

equation for a given model. Then, one may suppose that in the course of cosmological evolution the Universe appears to be in the state with $E = 0$, where the correlations of physical fields with the condensate are minimal. In this state the Universe in a large scale is described by general relativity.

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